

1. Consider an LTI system with frequency response

$$H(e^{j\omega}) = e^{-j(2\omega - \pi/4)} \left( \frac{1 + e^{-j\omega}}{1 + 0.5e^{-j\omega}} \right)$$



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Determine  $y[n]$ , the output of this system, if

$$x[n] = \cos\left(\frac{\pi n}{4}\right)$$

for all  $n$ . Your answer should contain a cos function.

$$\cos\left(\frac{9\pi n}{4}\right) = \frac{e^{j\frac{9\pi}{4}} + e^{-j\frac{9\pi}{4}}}{2}$$

$\frac{\pi}{4} = \dots$  down to  $\frac{\pi}{4}$   
and  $-\frac{\pi}{4}$

$$\begin{aligned} H(e^{j\frac{\pi}{4}}) &= e^{-j(2(\frac{\pi}{4}) - \pi/4)} \left( \frac{1 + e^{-j4(\frac{\pi}{4})} + 4e^{-j2(\frac{\pi}{4})}}{1 + 0.5e^{-j4(\frac{\pi}{4})}} \right) \\ &= e^{-j\pi/4} \left( \frac{1 + e^{-j\pi} + 4e^{-j2\pi}}{1 + 0.5e^{-j\pi}} \right) = e^{-j\pi/4} \left( \frac{1 - 1 + 4}{1 - 0.5} \right) \\ &= e^{-j\pi/4} \left( \frac{4}{1.5} \right) = 8e^{-j\pi/4} \end{aligned}$$

$$\begin{aligned} H(e^{j3\pi/4}) &= e^{-j(2(3\pi/4) - \pi/4)} \left( \frac{1 + e^{-j4(3\pi/4)} + 4e^{-j2(3\pi/4)}}{1 + 0.5e^{-j4(3\pi/4)}} \right) \\ &= e^{+j3\pi/4} \left( \frac{1 - 1 + 4}{1 - 0.5} \right) = 8e^{+j3\pi/4} \end{aligned}$$

$$\begin{aligned} y[n] &= \frac{8(e^{-j\pi/4} e^{j9\pi/4n} + e^{j3\pi/4} e^{-j9\pi/4n})}{2} \\ &= \frac{8(e^{-j\pi/4} e^{j\pi/2} e^{j\pi/4n} + e^{j\pi/2} e^{j\pi/4} e^{-j\pi/4n})}{2} \end{aligned}$$

$$y[n] = 8e^{+j\pi/4} \cos\left(\frac{\pi}{4}n - \frac{\pi}{2}\right)$$